NAG Fortran Library Routine Document F08UTF (CPBSTF/ZPBSTF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

F08UTF (CPBSTF/ZPBSTF) computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix.

2 Specification

```
SUBROUTINE FO8UTF(UPLO, N, KB, BB, LDBB, INFO)
ENTRY cpbstf (UPLO, N, KB, BB, LDBB, INFO)
INTEGER N, KB, LDBB, INFO
complex BB(LDBB,*)
CHARACTER*1 UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine computes a split Cholesky factorization of a complex Hermitian positive-definite band matrix B. It is designed to be used in conjunction with F08USF (CHBGST/ZHBGST).

The factorization has the form $B = S^H S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first (n+k)/2 rows, and transposed — hence, lower triangular — in the remaining rows. For example, if n=9 and k=2, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ & s_{22} & s_{23} & s_{24} \\ & s_{33} & s_{34} & s_{35} \\ & & s_{44} & s_{45} \\ & & s_{55} \\ & & s_{64} & s_{65} & s_{66} \\ & & s_{75} & s_{76} & s_{77} \\ & & & s_{86} & s_{87} & s_{88} \\ & & & s_{97} & s_{98} & s_{99} \end{pmatrix}$$

4 References

None.

5 Parameters

1: UPLO – CHARACTER*1

Input

On entry: indicates whether the upper or lower triangular part of B is stored as follows:

if UPLO = 'U', the upper triangular part of B is stored;

if UPLO = 'L', the lower triangular part of B is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER Input

On entry: n, the order of the matrix B.

Constraint: $N \ge 0$.

3: KB – INTEGER Input

On entry: k, the number of super-diagonals of the matrix B if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'.

Constraint: $KB \ge 0$.

4: BB(LDBB,*) – *complex* array

Input/Output

Note: the second dimension of the array BB must be at least max(1, N).

On entry: the n by n Hermitian positive-definite band matrix B, stored in rows 1 to k+1. More precisely, if UPLO = 'U', the elements of the upper triangle of B within the band must be stored with element b_{ij} in BB(k+1+i-j,j) for max $(1,j-k) \le i \le j$; if UPLO = 'L', the elements of the lower triangle of B within the band must be stored with element b_{ij} in BB(1+i-j,j) for $j \le i \le \min(n,j+k)$.

On exit: B is overwritten by the elements of its split Cholesky factor S.

5: LDBB – INTEGER Input

On entry: the first dimension of the array BB as declared in the (sub)program from which F08UTF (CPBSTF/ZPBSTF) is called.

Constraint: LDBB \geq KB + 1.

6: INFO – INTEGER Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = -i, the *i*th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i, the factorization could not be completed, because the updated element b_{ii} would be the square root of a negative number. Hence B is not positive-definite. This may indicate an error in forming the matrix B.

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix B+E, where

$$|E| < c(k+1)\varepsilon|S^H||S|,$$

c(k+1) is a modest linear function of k+1, and ε is the **machine precision**. It follows that $|e_{ij}| \le c(k+1)\varepsilon\sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $4n(k+1)^2$, assuming $n \gg k$.

A call to this routine may be followed by a call to F08USF (CHBGST/ZHBGST) to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive-definite.

The real analogue of this routine is F08UFF (SPBSTF/DPBSTF).

9 Example

See Section 9 of the document for F08USF (CHBGST/ZHBGST).